

SCATTERING PARAMETERS OF AXIAL INDUCTIVE STRIPS  
IN RECTANGULAR WAVEGUIDE

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## Abstract

A variational formula is derived for the reflection coefficient of axial inductive strips in rectangular waveguide. With a carefully chosen current basis function incorporating the edge condition, the computed results are in good agreement with the measurement.

## Introduction

An axial inductive strip is formed by inserting a metal strip in a rectangular waveguide such that the strip surface is parallel to the narrow waveguide wall (i.e., parallel to the E plane). It is a common circuit element in the design of planar circuits developed either on a metal sheet [1] or on copper-coated substrates [2]. For example, high-Q bandpass filters with the inductive strips have been successfully designed at microwave and millimeter-wave frequencies [3,4,5].

The circuit parameters of the axial strip have been studied by several authors [3,5,6,7]. In [5] and [7] the equivalent circuit reactances are obtained with variational methods and in [3] and [6] the scattering parameters are calculated by the residue-calculus technique and by a mode-matching method. Among the various methods, the one by Chang and Khan [7] is the only one that does not require complex matrix manipulation and thus minimizes the numerical efforts. Their result, however, is valid only for narrow strips because of the assumed constant current on the strip. It is, therefore, the purpose of this work to extend their approach by removing the narrow-strip limitation. To this end, a variational formula is derived for the reflection coefficient of the axial strip. With a carefully chosen current basis function incorporating the edge condition, the computed results are in good agreement with the measurements for both narrow and wide strips.

## Formulation

The structure being studied here is shown in Fig. 1. The strip is assumed to be perfectly conducting (as is the waveguide) and infinitesimally thin. The dielectric substrate, if any, is assumed to be lossless.

Consider a  $TE_{10}$  dominant-mode incident field. Since both the field distribution and the discon-

tinuity structure have no variation in the y direction, the scattered field will also have no variation in y and lie only in the y direction. The scattered field is then expressed by an integral equation in terms of the current distribution,  $J(z)$ , on the strip. Applying the boundary condition on the surface of the strip, we obtain an expression for the reflection coefficient,  $R$ , defined at  $z = 0$ :

$$R = \frac{-\frac{\phi_1^2(x_1)}{\gamma_1} \left[ \int_0^w J(z) e^{-\gamma_1 z} dz \right]^2}{\sum_{n=1}^{\infty} \frac{\phi_n^2(x_1)}{\gamma_n} \int_0^w \int_0^w J(z) J(z') e^{-\gamma_n |z-z'|} dz dz'} \quad (1)$$

where  $\phi_n(x)$  is the transverse field distribution and  $\gamma_n$  the propagation constant of n-th mode.  $x_1$  is the location of the strip and  $w$  the strip width. Using a method similar to Lewin [8], this expression is shown to be stationary for small variations in  $J(z)$  about its correct value. The same formula can also be derived by using the reaction concept [9].

With the expression in (1), the reflection coefficient can be evaluated using an approximate current distribution. If a constant current is used, we obtain the same results as in [7], valid for narrow strips. For wide strips, a more appropriate current distribution must be used to obtain accurate results. Considering the evanescent field behavior in the guide section containing the strip and incorporating the edge condition at  $z=0$ , we choose the following current distribution,

$J(z) = e^{-\gamma z} / \sqrt{z}$ , where  $\gamma$  is chosen to be the propagation constant of the fundamental mode in the evanescent guide section. Substituting this into (1), we have

$$R = \frac{-\frac{\phi_1^2(x_1)}{\gamma_1} \cdot \left[ \frac{1}{\sqrt{w}} \sum_{k=1}^{\infty} (-1)^{k-1} \frac{[(\gamma_1 + \gamma)w]^{k-1}}{(2k-1)(k-1)!} \right]^2}{\sum_{n=1}^{\infty} \frac{\phi_n^2(x_1)}{\gamma_n} \frac{2}{\gamma_n + \gamma} \left\{ \sum_{k=1}^{\infty} \frac{1}{2k-1} \left( \frac{\gamma_n - \gamma}{\gamma_n + \gamma} \right)^{k-1} \cdot \left[ 1 - e^{-(\gamma_n + \gamma)w} \sum_{m=0}^k \frac{[(\gamma_n + \gamma)w]^m}{m!} \right] \right\}} \quad (2)$$

Although the evaluation of (2) involves a double infinite summation, the summation with index  $k$  converges very fast.

The edge condition at the other end of the strip (i.e., at  $z = w$ ) can also be included in the current distribution, however, this leads to an expression for  $R$  with a slowly converging summation, especially when the strip width,  $w$ , is large. (In

fact, it sometimes diverges due to numerical errors.) In practice, when  $w$  is large, the contribution of the current near  $z = w$  to the reflection coefficient is negligible. Therefore, the edge condition at  $z = w$  is dropped.

### Results

To demonstrate the use of eqn. (1), we have computed the reflection coefficient for a metal sheet inserted in a Ka-band waveguide. Both the uniform current distribution and that with a single edge condition are used. In addition, the same structure is also analyzed with a mode-matching method [6]. The results are shown in Fig. 2 at a chosen frequency of 30 GHz. It shows that the result with the uniform current is good up to about 1 mm in this case. While the result with eqn. (2) is in good agreement for the full range with that obtained by mode-matching method.

The result is further verified by experiments. Two-mil thick copper strips are cut and inserted into an X-band test fixture. The scattering parameters are then measured across the full band width (8-12 GHz) with an automated network analyzer system at the Microwave Lab., Naval Postgraduate School. The resulting magnitude and phase of the reflection coefficients are shown in Fig. 3, in comparison with the theoretical data by eqn. (2) and by the mode-matching method. Good agreement is observed.

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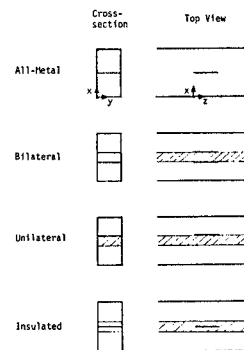


Fig.1 Geometry of axial inductive strips.

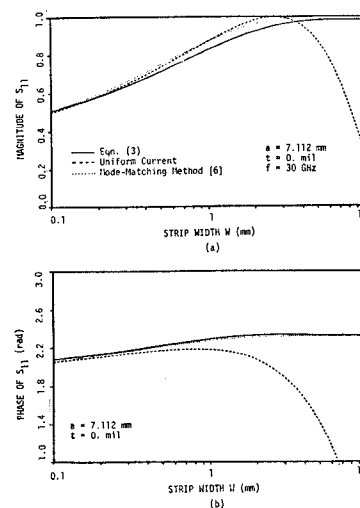


Fig.2 Magnitude and phase of the reflection coefficient as a function of strip width at 30 GHz.

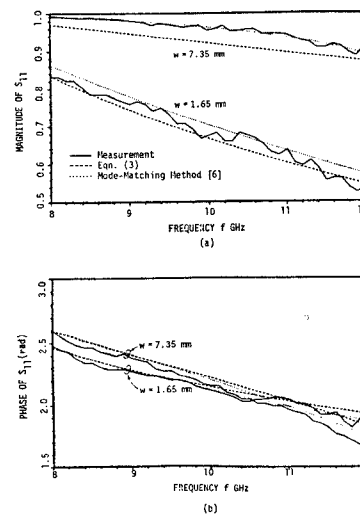


Fig.3 Magnitude and phase of the reflection coefficient as a function of frequency.